A Model of Depth and Water Equivalent of Snow in an Extensive Area

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1. INTRODUCTION
To estimate amounts of snow is necessary for the decision of routing and designing roads in heavy snow areas. The model capable of estimating snow amounts has been developed. The amounts of snowfall and snowmelt are estimated first and then water equivalent of snow is calculated. The model can provide the estimates in an extensive area by using a digital elevation model (DEM). Based on the densification theory of snow, the depth of snow can be calculated by the estimates of snowfall and snowmelt. Since the model needs only published meteorological data and DEM data, it can provide the estimates at any place and any date. Therefore the model can provide the estimates for long years and can be used for the study of extremes and return periods of snow amounts.

2. DESCRIPTION OF THE MODEL
2.1 Snowfall amount
Relation between probability occurrence of snowfall and air temperature is shown in Fig.1. Tsnow100% is defined at the air temperature below which rain does not fall. Tsnow0% is defined at the air temperature above which snow does not fall. When daily mean temperature is between them, the snowfall amount is assumed to be proportional to the difference between the daily mean temperature and Tsnow0%. When the daily mean temperature is, for example, half of Tsnow100% and Tsnow0%, a half amount of precipitation is snowfall. Analysis of meteorological data, whose

Fig. 1. Relation between probability occurrence of snowfall and air temperature
mean air temperature are between $T_{\text{snow100\%}}$ and $T_{\text{snow0\%}}$, has shown that the number of
days received both solid and liquid precipitation is more than the number of days received
either of them. Therefore the proportional method is adopted in the model. Daily amount of
precipitation and daily mean air temperature can give the amount of snowfall.
Measured precipitation amount by a rain gauge is generally less than true amount. Because it
disturbs the air current around itself, the gauge can catch less precipitation. The catch ratios of
rain gauges can be calculated based on the types of the gauge and wind speeds (Ohno et al.,
1998). In this model, the catch ratio is used to correct the measured snowfall amount.

2.2 Snowmelt
1) Simplified heat balance method
The energy received by the entire snow cover through the snow surface and bottom is given
as follows.

$$Ms + Mg = NRS + NRL + H + lE + R + Qi + G,$$ (1)

where $Ms$ is the energy for melt on the surface, $Mg$ is the energy for melt on the bottom, $NRS$
is net short-wave radiation, $NRL$ is net long-wave radiation, $H$ is sensible heat flux , $lE$
is latent heat flux, $R$ is the heat flux due to rain, $Qi$ is the change of internal energy and $G$ is the
conductive heat flux from the ground.

To calculate $NRL, H$ and $lE$ is not simple at all. Cloudiness and vapour pressure are necessary
for the calculation of $NRL$. Wind speed is necessary for $H$ and $lE$. Those meteorological
elements are difficult to estimate at non-observation places. Suizu (2001) introduced a
simplified heat flux method and calculated $NRL, H$ and $lE$ by precipitation, air temperature
and sunshine duration.

According to the method, $NRL$ is calculated by the rate of sunshine ($= $ sunshine duration $N /
possible duration of sunshine $N_0$). The relation between $NRL$ and $N / N_0$ is given by

$$NRL = -13.7 N / N_0 - 3.0.$$ (2)

In this paper the unit of energy is expressed in mm converted to the depth of snowmelt. The
relation was deduced from the snow and radiation data during snow cover seasons in
northern and central Japan.

$H$ and $lE$ are deducted from the bulk formulas. The bulk coefficients for sensible and latent
heat are almost same, so

$$H = K_{SL} \cdot Ta \cdot P / 1013$$ (3)

$$lE = 1.53 \cdot K_{SL} \cdot (e - 6.11)$$ (4)

where $K_{SL}$ is introduced by Suizu (2001), $Ta$ (°C) is daily mean air temperature, $P$ (mb) is
atmospheric pressure and $e_s$ (mb) is the saturation vapor pressure to $Ta$. Equation (3) is
applied only when $Ta$ is more than 0 °C. Equation (4) is applied only on the rainy day with
more than 7 °C of $Ta$. Snowmelt due to latent heat flux does not occur on cold or dry days,
because the vapor pressure in air is lower than that on the snow surface. Therefore the
condition for the latent heat is necessary. The condition for Equation (4) was deduced from
the data observed during melting seasons in northern and central Japan.
Other factors except NRL, $H$ and $IE$ are calculated by existing methods. Daily snowmelt can be calculated by precipitation, air temperature, sunshine duration and $K_{SL}$ with the simplified heat balance method. Those three meteorological elements are measured at the stations of Automated Meteorological Data Acquisition System (AMeDAS) in Japan. Each station covers around 21 km square. Snow depth is measured at the stations in heavy snow region.

2) Determination of $K_{SL}$

$K_{SL}$ is a function of the bulk coefficients and wind speed. $K_{SL}$ cannot be determined directly without any difficulty of the estimation of wind speed. Therefore $K_{SL}$ is determined indirectly by using the data of snow depth. Total amounts of snowfall and snowmelt until the day when snow cover melts away, should be same. An example of the determination is shown in Fig. 2. Snowfall amount is computed based on air temperature and precipitation. Snowmelt except for sensible and latent heat is computed based on precipitation, air temperature, sunshine duration. If $K_{SL}$ is changed, the daily snowmelt changes, so does the date of melting away. If computed date of melting away falls on the actual date, $K_{SL}$ is correctly determined.

3) Relationship between $K_{SL}$ and topography

$K_{SL}$ have been determined at many stations. It shows that $K_{SL}$ is a function of not only wind speed but also the diurnal variation of wind speed. Where wind blows strongly in the afternoon, the maximum of air temperature falls on the maximum of wind speed. Consequently $H$, $IE$ and $K_{SL}$ become larger. This tendency is shown at the stations in basins. $K_{SL}$ is larger near the sea or on ridges because of strong wind. $K_{SL}$ is also larger in basins because of diurnal wind variation. It suggests that $K_{SL}$ can be determined by the topography of station.

2.3 Water equivalent and depth of snow

The reputation of calculations of snowfall and snowmelt provides the water equivalent of snow daily.

Kojima (1967) studied the densification of snow by using a viscous compression theory and obtained the equation to compute depth-density profiles numerically for seasonal snow. Motoyama (1990) used the equation and the temperature-index method for snowmelt and simulated the depth of snow at several meteorological stations in Japan. The depth of snow
can be computed by using daily snowfall, snowmelt and some coefficient of the densification of snow.

2.4 Calculation of the distribution of precipitation, air temperature and sunshine hour

Daily snowfall, snowmelt, water equivalent and depth of snow can be calculated at any place based on the estimates of precipitation, air temperature and sunshine duration at the place. Those meteorological elements at any place are calculated by topographic factors and meteorological data of AMeDAS. Topographic factors are such as elevation, distance from the sea, concavity or convexity, inclination, and so on which express topographic features. The topographic factors should be expressed in numerical values and can be calculated by using of a DEM.

The procedure of estimating air temperature at any place is shown in Fig.3. First differences of monthly precipitations of a station to those of others are calculated. Second a regression equation of the difference is derived, as multiple regression analysis is applied to the differences as criterion variables and the topographic factors as explanatory variables. Third the estimate from each station can be obtained, as the topographic factors of any place are substituted for the regression equation and the observed value of air temperature is added. As each station can provide an estimate, the number of the estimates is as many as that of the stations. Finally the estimates are unified through the interpolation to the inverse of distance between the place and each station.

The procedure to estimate precipitation and sunshine duration are almost same and not the differences but their ratios are used. Dependence on the topographic factors varies seasonally, so the regression equations are made monthly. $K_{SL}$ is determined by the regression equation with the topographic factors.

2.5 Calculation of inclination and shading from the sun with topographic features

Net short-wave radiation on a level plane can be obtained by using sunshine duration. Real snow surface is on slope and it can be shaded from the sun with surrounding topography. The inclination, the azimuth and elevation of skyline can be calculated by a DEM at any place. The effect of the slope and shading on snow surface is calculated based on the inclination, the skyline and the azimuth and elevation of the sun. Then the net short-wave radiation on real surface is computed.
3. APPLICATION AND VERIFICATION OF THE MODEL

3.1 Applied area
The area, where the model was applied, and the locations of AMeDAS stations are shown in Fig. 4. The area is near the Sea of Japan and the range of elevation is from 0 to 3000m. Sasagamine is not an AMeDAS station but air temperature, solar radiation and the depth and weight of snow are measured.

The DEM, which give elevations at the lattices of 250 and 50m apart, was used for the computation. The DEM is published by the Geographical Survey Institute of Japan.

3.2 Verification of snowfall amounts and air temperature

Data of Sasagamine were used in the multiple regression analysis but not used for daily calculation. Fig. 5 and 6 show the relation between measured and estimated meteorological elements. The

<table>
<thead>
<tr>
<th></th>
<th>air temperature</th>
<th>solar radiation</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (measured)</td>
<td>-0.81 °C</td>
<td>10.4 MJ·m⁻²·d⁻¹</td>
</tr>
<tr>
<td>mean (computed)</td>
<td>-0.74 °C</td>
<td>10.7 MJ·m⁻²·d⁻¹</td>
</tr>
<tr>
<td>root mean square error</td>
<td>1.57 °C</td>
<td>2.7 MJ·m⁻²·d⁻¹</td>
</tr>
<tr>
<td>correlation coefficient</td>
<td>0.97</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Fig. 4. Location map, showing elevation and AMeDAS stations.

Fig. 5. Measured vs estimated air temperature at Sasagamine.

Fig. 6. Measured vs estimated air temperature at Sasagamine.
of the estimates is shown in Table 1. According to the results of the calculation, the estimates of both air temperature and solar radiation have high accuracy.

3.3 Verification of water equivalent and depth of snow

As an example the distributions of water equivalent of snow at beginning of every month in 1995-96 snow season are shown in Fig. 7. The figure shows that the snow has melted away in lower elevation but the snow increases in higher elevation even until May. The distribution at any date is obtainable by this model. The time series of calculated depth and water equivalent of snow at Sasagamine are shown with measured values in Fig. 8 and 9 respectively. Both the depth and the water equivalent are good accord to the measured and it shows the model has high accuracy.

4. CONCLUSION

The model, which can provide the time series and spatial distribution of depth and water equivalent of snow, has been developed. The verification of the model shows it has high accuracy. All inputting data to the model are published data, meteorological and DEM data, in Japan. Those data are published by governmental agencies and can be purchased by anyone. Therefore the model can provide the estimates at any place and date in Japan. The model can be applied to wherever those data are available. AMeDAS data have been collected and published by Meteorological Agency of Japan since 1976. Consequently the model can provide the depth and water equivalent of snow in a long term only by using the data published. The long term calculation by this model is well applicable.

Fig. 7. Distributions of water equivalent of snow
to the study of the extremes and return periods of snow.

Fig. 8. Time series of water equivalent of snow at Sasagamine

Fig. 9. Time series of snow depth at Sasagamine
REFERENCES


